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A-LEVEL
FURTHER MATHEMATICS
(7367)

Specification
For teaching from September 2017 onwards
For exams in June 2019 onwards

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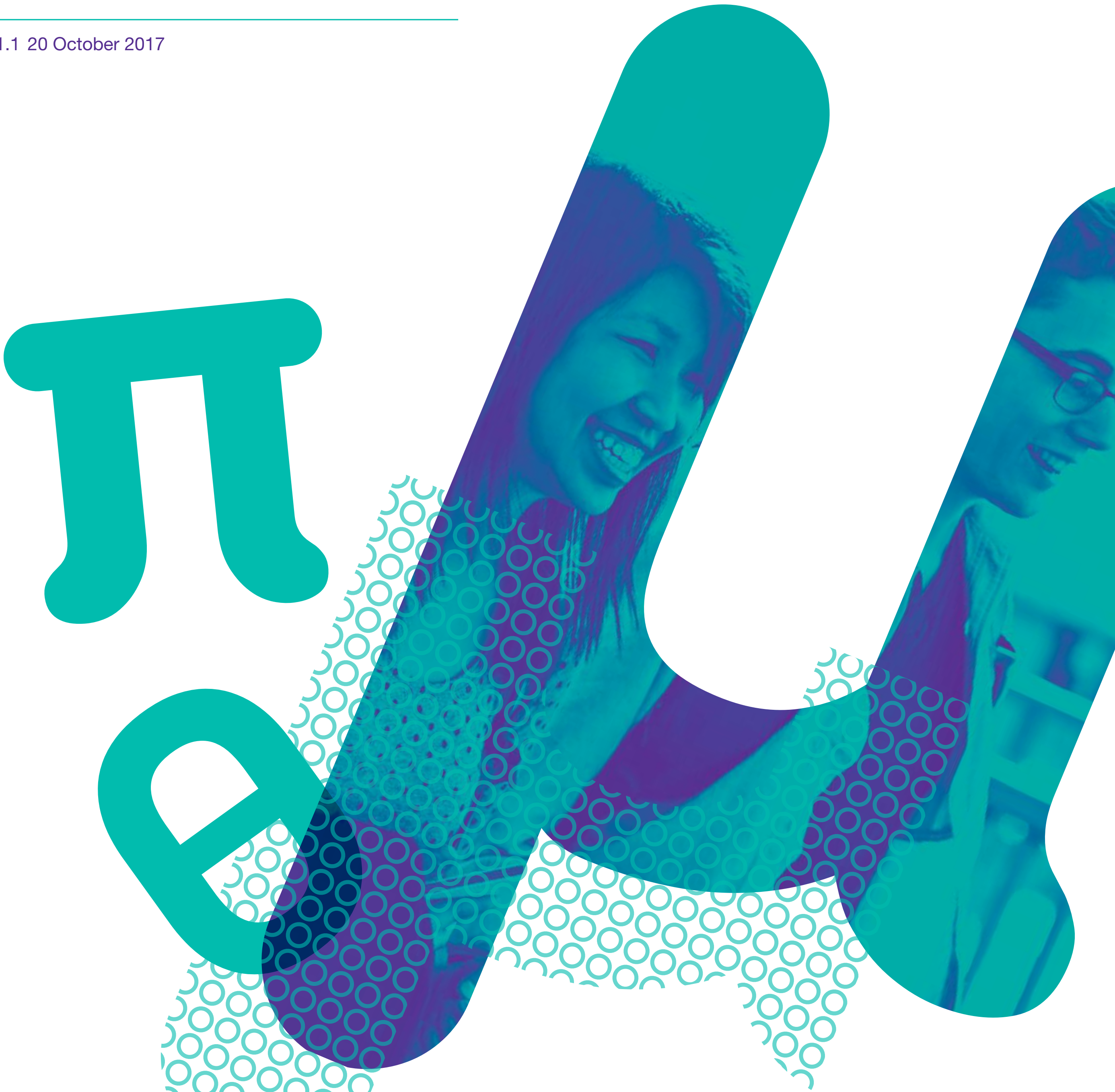

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1 Introduction

1.1 Why choose AQA for A-level Further Mathematics

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2 Specification at a glance

This qualification is linear. Linear means that students will sit all their exams at the end of the course.

This A-level qualification builds on the skills, knowledge and understanding set out in the whole GCSE (9–1) subject content for mathematics and the subject content for AS and A-level mathematics.

2.1 Subject content

All students must study this core content.

- OT1: Mathematical argument, language and proof (page 11)
- OT2: Mathematical problem solving (page 11)
- OT3: Mathematical modelling (page 12)
- Compulsory content (page 12)

Students must study two of these options.

- Optional application 1 – mechanics (page 20)
- Optional application 2 – statistics (page 23)
- Optional application 3 – discrete mathematics (page 27)

2.2 Assessments

Paper 1
What's assessed May assess content from the following sections: • A: Proof • B: Complex numbers • C: Matrices • D: Further algebra and functions • E: Further calculus • F: Further vectors • G: Polar coordinates • H: Hyperbolic functions • I: Differential equations • J: Numerical methods
How it's assessed • Written exam: 2 hours • 100 marks • 33⅓ % of A-level
Questions A mix of question styles, from short, single-mark questions to multi-step problems.

Paper 2

What's assessed

May assess content from the following sections:

- A: Proof
- B: Complex numbers
- C: Matrices
- D: Further algebra and functions
- E: Further calculus
- F: Further vectors
- G: Polar coordinates
- H: Hyperbolic functions
- I: Differential equations
- J: Numerical methods

How it's assessed

- Written exam: 2 hours
- 100 marks
- 33⅓ % of A-level

Questions

A mix of question styles, from short, single-mark questions to multi-step problems.

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Paper 3

What's assessed

One question paper answer booklet on Discrete and one question paper answer booklet on Statistics.

How it's assessed

- Written exam: 2 hours
- 100 marks
- 33⅓ % of A-level

Questions

A mix of question styles, from short, single-mark questions to multi-step problems.

OR

Paper 3
What's assessed One question paper answer booklet on Statistics and one question paper answer booklet on Mechanics.
How it's assessed • Written exam: 2 hours • 100 marks • 33⅓ % of A-level
Questions A mix of question styles, from short, single-mark questions to multi-step problems.

OR

Paper 3
What's assessed One question paper answer booklet on Mechanics and one question paper answer booklet on Discrete.
How it's assessed • Written exam: 2 hours • 100 marks • 33⅓ % of A-level
Questions A mix of question styles, from short, single-mark questions to multi-step problems.

3 Subject content

The subject content in sections A to J is compulsory for all students. Students must study two of the optional applications. The optional applications are mechanics (MA to ME), statistics (SA to SH) and discrete (DA to DG).

3.1 Overarching themes

A-level specifications in further mathematics must require students to demonstrate the overarching knowledge and skills contained in sections **OT1**, **OT2** and **OT3**. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out in sections **A** to **DG**.

Appendix A sets out the mathematical notation that students are required to understand for this qualification. Appendix B sets out the mathematical formulae and identities students are required to use in this qualification. Further information is provided in the appendices.

3.1.1 OT1: Mathematical argument, language and proof

	Content
OT1.1	Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable.
OT1.2	Understand and use mathematical language and syntax as set out in the content.
OT1.3	Understand and use language and symbols associated with set theory, as set out in the content.
OT1.4	Understand and use the definition of a function; domain and range of functions.
OT1.5	Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics.

3.1.2 OT2: Mathematical problem solving

	Content
OT2.1	Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved.
OT2.2	Construct extended arguments to solve problems presented in an unstructured form, including problems in context.
OT2.3	Interpret and communicate solutions in the context of the original problem.

	Content
OT2.6	Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle.
OT2.7	Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics.

3.1.3 OT3: Mathematical modelling

	Knowledge/skill
OT3.1	Translate a situation in context into a mathematical model, making simplifying assumptions.
OT3.2	Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student).
OT3.3	Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student).
OT3.4	Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate.
OT3.5	Understand and use modelling assumptions.

3.2 Compulsory content

3.2.1 A: Proof

	Content
A1	Construct proofs using mathematical induction; contexts include sums of series, divisibility, and powers of matrices.

3.2.2 B: Complex numbers

	Content
B1	Solve any quadratic equation with real coefficients; solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics).

	Content
B2	Add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$ with x and y real; understand and use the terms 'real part' and 'imaginary part'.

	Content
B3	Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.

	Knowledge/skill
B4	Use and interpret Argand diagrams.

	Content
B5	Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed).

	Content
B6	Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed).

	Content
B7	Construct and interpret simple loci in the Argand diagram such as $\lvert z - a \rvert > r$ and $\arg(z - a) = \theta$ (knowledge of radians is assumed).

	Knowledge/skill
B8	Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.

	Content
B9	Know and use the definition $e^{i\theta} = \cos\theta + i\sin\theta$ and the form $z = re^{i\theta}$

	Content
B10	Find the n distinct n th roots of $re^{i\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n -gon in the Argand diagram.

	Content
B11	Use complex roots of unity to solve geometric problems.

3.2.3 C: Matrices

	Content
C1	Add, subtract and multiply conformable matrices; multiply a matrix by a scalar.

	Content
C2	Understand and use zero and identity matrices.

	Content
C3	Use matrices to represent linear transformations in 2D; successive transformations; single transformations in 3D (3D transformations confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes) (knowledge of 3D vectors is assumed).

	Content
C4	Find invariant points and lines for a linear transformation.

	Content
C5	Calculate determinants of 2 × 2 and 3 × 3 matrices and interpret as scale factors, including the effect on orientation.

	Content
C6	Understand and use singular and non-singular matrices; properties of inverse matrices. Calculate and use the inverse of non-singular 2 × 2 matrices and 3 × 3 matrices.

	Content
C7	Solve three linear simultaneous equations in three variables by use of the inverse matrix.

	Content
C8	Interpret geometrically the solution and failure of solution of three simultaneous linear equations.

	Content
C9	Factorisation of determinants using row and column operations.

	Content
C10	Find eigenvalues and eigenvectors of 2 × 2 and 3 × 3 matrices. Find and use the characteristic equation. Understand the geometrical significance of eigenvalues and eigenvectors.

	Content
C11	Diagonalisation of matrices; $\mathbf{M} = \mathbf{UDU}^{-1}$; $\mathbf{M}^{n} = \mathbf{UD}^{n}\mathbf{U}^{-1}$; when eigenvalues are real.

3.2.4 D: Further algebra and functions

	Content
D1	Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations.

	Content
D2	Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).

	Content
D3	Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series.

	Content
D4	Understand and use the method of differences for summation of series including use of partial fractions.

	Content
D5	Find the Maclaurin series of a function including the general term.

	Content
D6	Recognise and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$, and $(1+x)^n$, and be aware of the range of values of x for which they are valid (proof not required).

	Content
D7	Evaluation of limits using Maclaurin series or l'Hôpital's rule.

	Content
D8	Inequalities involving polynomial equations (cubic and quartic).

	Content
D9	Solving inequalities such as $\frac{ax+b}{cx+d} < ex + f$ algebraically.

	Content
D10	Modulus of functions and associated inequalities.

	Content
D11	Graphs of $y = \lvert \mathrm{f}(x) \rvert$, $y = \frac{1}{\mathrm{f}(x)}$ for given $y = \mathrm{f}(x)$

	Content
D12	Graphs of rational functions of form $\frac{ax+b}{cx+d}$; asymptotes, points of intersection with coordinate axes or other straight lines; associated inequalities.

	Content
D13	Graphs of rational functions of form $\frac{ax^2+bx+c}{dx^2+ex+f}$, including cases when some of these coefficients are zero; asymptotes parallel to coordinate axes; oblique asymptotes.

	Content
D14	Using quadratic theory (not calculus) to find the possible values of the function and coordinates of the stationary points of the graph for rational functions of form $\frac{ax^2+bx+c}{dx^2+ex+f}$

	Content
D15	Sketching graphs of curves with equations $y^2 = 4ax$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $xy = c^2$ including intercepts with axes and equations of asymptotes of hyperbolas.

	Content
D16	Single transformations of curves involving translations, stretches parallel to coordinate axes and reflections in the coordinate axes and the lines $y = \pm x$. Extend to composite transformations including rotations and enlargements.

3.2.5 E: Further calculus

	Content
E1	Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.

	Content
E2	Derive formulae for and calculate volumes of revolution.

	Content
E3	Understand and evaluate the mean value of a function.

	Content
E4	Integrate using partial fractions (extend to quadratic factors $ax^2 + c$ in the denominator).

	Content
E5	Differentiate inverse trigonometric functions.

	Content
E6	Integrate functions of the form $\left(a^2 - x^2\right)^{-\frac{1}{2}}$ and $\left(a^2 + x^2\right)^{-1}$ and be able to choose trigonometric substitutions to integrate associated functions.

	Content
E7	Arc length and area of surface of revolution for curves expressed in Cartesian or parametric coordinates.

	Content
E8	Derivation and use of reduction formulae for integration.

	Content
E9	The limits $\lim\limits_{x \to \infty}\left(x^k e^{-x}\right)$ and $\lim\limits_{x \to 0}\left(x^k \ln x\right)$ where $k > 0$, applied to improper integrals

3.2.6 F: Further vectors

	Content
F1	Understand and use the vector and Cartesian forms of an equation of a straight line in 3D.

	Content
F2	Understand and use the vector and Cartesian forms of the equation of a plane.

	Content
F3	Calculate the scalar product and use it to calculate the angle between two lines, to express the equation of a plane, and to calculate the angle between two planes and the angle between a line and a plane.

	Content
F4	Check whether vectors are perpendicular by using the scalar product.

	Content
F5	Calculate and understand the properties of the vector product. Understand and use the equation of a straight line in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$. Use vector products to find the area of a triangle.

	Content
F6	Find the intersection of two lines. Find the intersection of a line and a plane. Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.

3.2.7 G: Polar coordinates

	Content
G1	Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates.

	Content
G2	Sketch curves with r given as a function of θ , including use of trigonometric functions.

	Content
G3	Find the area enclosed by a polar curve.

3.2.8 H: Hyperbolic functions

	Content
H1	Understand the definitions of hyperbolic functions sinh x , cosh x and tanh x , including their domains and ranges, and be able to sketch their graphs. Understand the definitions of hyperbolic functions sech x , cosech x and coth x , including their domains and ranges.

	Content
H2	Differentiate and integrate hyperbolic functions.

	Content
H3	Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.

	Content
H4	Derive and use the logarithmic forms of the inverse hyperbolic functions.

	Content
H5	Integrate functions of the form $\left(x^2+a^2\right)^{-\frac{1}{2}}$ and $\left(x^2-a^2\right)^{-\frac{1}{2}}$ and be able to choose substitutions to integrate associated functions.

	Content
H6	Understand and use $\tanh x \equiv \frac{\sinh x}{\cosh x}$ Understand and use $\cosh^2 x - \sinh^2 x \equiv 1$; $\text{sech}^2 x \equiv 1 - \tanh^2 x$ and $\text{cosech}^2 x \equiv \coth^2 x - 1$, $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$, $\sinh 2x \equiv 2\sinh x \cosh x$

	Content
H7	Construct proofs involving hyperbolic functions and identities.

3.2.9 I: Differential equations

	Content
I1	Find and use an integrating factor to solve differential equations of the form $\frac{dy}{dx} + P(x)y = Q(x)$ and recognise when it is appropriate to do so.

	Content
I2	Find both general and particular solutions of differential equations.

	Content
I3	Use differential equations in modelling in kinematics and in other contexts.

	Content
I4	Solve differential equations of the form $y'' + ay' + by = 0$ where a and b are constants, by using the auxiliary equation.

	Content
I5	Solve differential equations of the form $y'' + ay' + by = f(x)$ where a and b are constants by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $f(x)$ is a polynomial, exponential or trigonometric function).

	Content
I6	Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation.

	Content
I7	Solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion.

	Content
I8	Model damped oscillations using 2nd order differential equations and interpret their solutions. Understand light, critical and heavy damping and be able to determine when each will occur.

	Content
I9	Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled 1st order simultaneous equations and be able to solve them, for example predator-prey models.

	Content
I10	Use of Hooke's Law with $T = kx$ to formulate a differential equation for simple harmonic motion, where k is a constant.

	Content
I11	Use models for damped motion where the damping force is proportional to the velocity.

3.2.10 J: Numerical methods

	Content
J1	Mid-ordinate rule and Simpson's rule for integration.

	Content
J2	Euler's step by step method for solving first order differential equations.

	Content
J3	Improved Euler method for solving first order differential equations. $y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r),\ x_{r+1} = x_r + h$

3.3 Optional application 1 – mechanics

3.3.1 MA: Dimensional analysis

	Content
MA1	Finding dimensions of quantities; checking for dimensional consistency.

	Content
MA2	Prediction of formulae; finding powers in potential formulae.

3.3.2 MB: Momentum and collisions

	Content
MB1	Conservation of momentum for linear motion and cases where velocities are given as one or two dimensional vectors (problems which require resolving).

	Content
MB2	Coefficient of restitution and Newton's Experimental Law. Use in direct collisions and impacts with a fixed smooth surface. Problems which require resolving.

	Content
MB3	Impulse and its relation to momentum. Use of $Ft = mv - mu$. Problems which require resolving.

	Content
MB4	Impulse for variable forces. One dimension only. Use of $I = \int F\mathrm{d}t$.

3.3.3 MC: Work, energy and power

	Content
MC1	Work done by a force acting in the direction of motion or directly opposing the motion. Use of $WD = Fd \cos\theta$

	Content
MC2	Gravitational potential energy. Use in conservation of energy problems.

	Content
MC3	Kinetic energy. Use in conservation of energy problems.

	Content
MC4	Hooke's Law including using modulus of elasticity. Use of $T = kx$ or $T = \frac{\lambda}{l}x$

	Content
MC5	Work done by a variable force. Use of $WD = \int F\mathrm{d}x$. Use in conservation of energy problems.

	Content
MC6	Elastic potential energy using modulus of elasticity. Use of $EPE = \frac{kx^2}{2}$ and $EPE = \frac{\lambda x^2}{2l}$. Use in conservation of energy problems.

	Content
MC7	Power. Use of $P = Fv$

3.3.4 MD: Circular motion

	Content
MD1	Motion of a particle moving in a circle with constant speed (knowledge of radians assumed).

	Content
MD2	Understand the definition of angular speed. Use both radians and revolutions per unit time.

	Content
MD3	Relationships between speed, angular speed, radius and acceleration. Use of $v = r\omega$, $a = r\omega^2$ and $a = \frac{v^2}{r}$

	Content
MD4	Use position, velocity and acceleration as vectors in the context of circular motion.

	Content
MD5	Conical pendulum, with one or two strings.

	Content
MD6	Circular motion in a vertical plane. Includes conditions to complete vertical circles. Use of conservation of energy in this context.

3.3.5 ME: Centres of mass and moments

	Content
ME1	Centre of mass for a system of particles.

	Content
ME2	Centre of mass for a composite body.

	Content
ME3	Centre of mass of a lamina by integration.

	Content
ME4	Centres of mass of bodies formed by rotating a region about the x -axis.

	Content
ME5	Conditions for sliding and toppling. Problems including suspension and on an inclined plane.

	Content
ME6	Determine the forces acting on a rigid body in equilibrium. Use of moments and couples.

3.4 Optional application 2 – statistics

3.4.1 SA: Discrete random variables (DRVs) and expectation

	Content
SA1	Understand DRVs with distributions given in the form of a table or function.

	Content
SA2	Evaluate probabilities for a DRV.

	Content
SA3	Evaluate measures of average and spread for a DRV to include mean, variance, standard deviation, mode and median.

	Content
SA4	Understand expectation and know the formulae: $E(X) = \sum x_i p_i$; $E(X^2) = \sum x_i^2 p_i$; $\text{Var}(X) = E(X^2) - (E(X))^2$

	Content
SA5	Understand expectation of linear functions of DRVs and know the formulae: $E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2\text{Var}(X)$ Know the formula $E(g(X)) = \sum g(x_i) p_i$ Find the mean, variance and standard deviation for functions of a DRV such as $E(5X^3)$, $E(18X^{-3})$, $\text{Var}(6X^{-1})$

	Content
SA6	Know the discrete uniform distribution defined on the set $\{1,2,\ldots,n\}$. Understand when this distribution can be used as a model.

	Content
SA7	Proof of mean and variance of discrete uniform distribution.

3.4.2 SB: Poisson distribution

	Content
SB1	Understand conditions for a Poisson distribution to model a situation. Understand terminology $X \sim \text{Po}(\lambda)$.

	Content
SB2	Know the Poisson formula and calculate Poisson probabilities using the formula or equivalent calculator function.

	Content
SB3	Know mean, variance and standard deviation of a Poisson distribution. Use the result that, if $X \sim \text{Po}(\lambda)$ then the mean and variance of X are equal.

	Content
SB4	Understand the distribution of the sum of independent Poisson distributions.

	Content
SB5	Formulate hypotheses and carry out a hypothesis test of a population mean from a single observation from a Poisson distribution using direct evaluation of Poisson probabilities.

3.4.3 SC: Type I and Type II errors

	Content
SC1	Understand Type I and Type II errors and define in context. Calculate the probability of making a Type I error from tests based on a Poisson or Binomial distribution. Calculate probability of making Type I error from tests based on a normal distribution.

	Content
SC2	Understand the power of a test. Calculations of P(Type II error) and power for a test for tests based on a normal, Binomial or a Poisson distribution.

3.4.4 SD: Continuous random variables (CRVs)

	Content
SD1	Understand and use a probability density function, $\mathrm{f}(x)$, for a continuous distribution and understand the differences between discrete and continuous distributions. Understand and use distributions of random variables that are part discrete and part continuous.

	Content
SD2	Find the probability of an observation lying in a specified interval.

	Content
SD3	Find the median and quartiles for a given probability density function, $\mathrm{f}(x)$.

	Content
SD4	Find the mean, variance and standard deviation for a given pdf, $\mathrm{f}(x)$. Know the formulae $\mathrm{E}(X) = \int x\mathrm{f}(x)\mathrm{d}x$, $\mathrm{E}(X^2) = \int x^2\mathrm{f}(x)\mathrm{d}x$, $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$

	Content
SD5	Understand the expectation and variance of linear functions of CRVs and know the formulae: $\mathrm{E}(aX+b) = a\mathrm{E}(X)+b$ and $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$ Know the formula $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{g}(x)\mathrm{f}(x)\mathrm{d}x$ Find the mean, variance and standard deviation of functions of a continuous random variable such as $\mathrm{E}(5X^3)$, $\mathrm{E}(18X^{-3})$, $\mathrm{Var}(6X^{-1})$

	Content
SD6	Understand and use a cumulative distribution function, $\mathrm{F}(x)$. Know the relationship between $\mathrm{f}(x)$ and $\mathrm{F}(x)$. $\mathrm{F}(x) = \int_{-\infty}^{x} \mathrm{f}(t)\mathrm{d}t$ and $\mathrm{f}(x) = \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{F}(x)$

	Content
SD7	Understand the rectangular distribution $\mathrm{f}(x)$ where $\mathrm{f}(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$ Know the conditions for the rectangular distribution to be used as a model. Calculate probabilities from a rectangular distribution. Know proofs of mean, variance and standard deviation for a rectangular distribution.

	Content
SD8	Know that if X and Y are independent (discrete or continuous) random variables then $\mathrm{E}(X + Y) = \mathrm{E}(X) + \mathrm{E}(Y)$ and $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$

3.4.5 SE: Chi squared tests for association

	Content
SE1	Construction of $n \times m$ contingency tables.

	Content
SE2	Use of $\sum \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2 statistic with appropriate degrees of freedom.

	Content
SE3	Know and use the convention that all E_i should be greater than 5.

	Content
SE4	Identification of sources of association in the context of a question.

	Content
SE5	Knowledge of when and how to apply Yates' correction.

3.4.6 SF: Exponential distribution

	Content
SF1	Know the conditions for an exponential distribution to be used as a model. Know the probability density function, $\mathrm{f}(x)$, and the cumulative distribution function, $\mathrm{F}(x)$, for an exponential distribution.

	Content
SF2	Calculate probabilities for an exponential distribution using $F(x)$ or integration of $f(x)$

	Content
SF3	Know proofs of mean, variance and standard deviation for an exponential distribution.

	Content
SF4	Understand that the lengths of intervals between Poisson events have an exponential distribution.

3.4.7 SG: Inference – one sample t - distribution

	Content
SG1	Test for the mean of a normal distribution with unknown variance using a t -statistic with appropriate degrees of freedom.

3.4.8 SH: Confidence Intervals

	Content
SH1	Construct symmetric confidence intervals for the mean of a normal distribution with known variance.

	Content
SH2	Construct symmetric confidence intervals from large samples, for the mean of a normal distribution with unknown variance.

	Content
SH3	Make inferences from constructed or given confidence intervals.

	Content
SH4	Construct symmetric confidence intervals from small samples, for the mean of a normal distribution with unknown variance using the t -distribution.

3.5 Optional application 3 – discrete mathematics

3.5.1 DA: Graphs

	Content
DA1	Understand and use the language of graphs including: vertex, edge, trail, cycle, connected, degree, subgraph, subdivision, multiple edge and loop.

	Content
DA2	Identify or prove properties of a graph including that a graph is Eulerian, semi-Eulerian or Hamiltonian.

	Content
DA3	Understand and use Euler's formula for connected planar graphs.

	Content
DA4	Use Kuratowski's Theorem to determine the planarity of graphs.

	Content
DA5	Understand and use complete graphs and bipartite graphs, including adjacency matrices and the complement of a graph.

	Content
DA6	Understand and use simple graphs, simple-connected graphs and trees.

	Content
DA7	Recognise and find isomorphism between graphs.

3.5.2 DB: Networks

	Content
DB1	Understand and use the language of networks including: node, arc and weight.

	Content
DB2	Solve network optimisation problems using spanning trees.

	Content
DB3	Solve route inspection problems.

	Content
DB4	Find and interpret upper bounds and lower bounds for the travelling salesperson problem.

	Content
DB5	Evaluate, modify and refine models which use networks.

3.5.3 DC: Network flows

	Content
DC1	Interpret flow problems represented by a network of directed arcs.

	Content
DC2	Find the value of a cut and understand its meaning.

	Content
DC3	Use and interpret the maximum flow-minimum cut theorem.

	Content
DC4	Introduce supersources and supersinks to a network.

	Content
DC5	Augment flows and determine the maximum flow in a network

	Content
DC6	Solve problems involving arcs with upper and lower capacities.

	Content
DC7	Refine network flow problems including using nodes of restricted capacity.

3.5.4 DD: Linear programming

	Content
DD1	Formulate constrained optimisation problems.

	Content
DD2	Solve constrained optimisation problems via graphical methods.

	Content
DD3	Use the Simplex algorithm for optimising (maximising and minimising) an objective function including the use of slack variables.

	Content
DD4	Interpret a Simplex tableau.

3.5.5 DE: Critical path analysis

	Content
DE1	Construct, represent and interpret a precedence (activity) network using activity-on-node.

	Content
DE2	Determine earliest and latest start and finish times for an activity network.

	Content
DE3	Identify critical activities, critical paths and the float of non-critical activities.

	Content
DE4	Refine models and understand the implications of possible changes in the context of critical path analysis.

	Content
DE5	Construct and interpret Gantt (cascade) diagrams and resource histograms.

	Content
DE6	Carry out resource levelling (using heuristic procedures) and solve problems where resources are restricted.

3.5.6 DF: Game theory for zero-sum games

	Content
DF1	Understand, interpret and construct pay-off matrices.

	Content
DF2	Find play-safe strategies and the value of the game.

	Content
DF3	Prove the existence or non-existence of a stable solution.

	Content
DF4	Identify and make use of dominated strategies.

	Content
DF5	Find optimal mixed strategies for a game including use of graphical methods.

	Content
DF6	Convert and solve higher order games to linear programming problems.

3.5.7 DG: Binary operations

	Content
DG1	Understand and use binary operations including use of modular arithmetic and matrix multiplication.

	Content
DG2	Understand, use and prove the commutativity of a binary operation.

	Content
DG3	Understand, use and prove the associativity of a binary operation.

	Content
DG4	Construct a Cayley table for a given set under a given binary operation.

	Content
DG5	Understand and prove the existence of an identity element for a given set under a given binary operation.

	Content
DG6	Find the inverse of an element belonging to a given set under a given binary operation.

	Content
DG7	Understand and use the language of groups including: order, period, subgroup, proper, trivial, non-trivial.

	Content
DG8	Understand and use the group axioms: closure, identity, inverses and associativity, including use of Cayley tables.

	Content
DG9	Recognise and use finite and infinite groups and their subgroups, including: groups of symmetries of regular polygons, cyclic groups and abelian groups.

	Content
DG10	Understand and use Lagrange's theorem.

	Content
DG11	Identify and use the generators of a group.

	Content
DG12	Recognise and find isomorphism between groups of finite order.

4 Scheme of assessment

Find past papers and mark schemes, and specimen papers for new courses, on our website at aqa.org.uk/pastpapers

This specification is designed to be taken over two years.

This is a linear qualification. In order to achieve the award, students must complete all assessments at the end of the course and in the same series.

A-level exams and certification for this specification are available for the first time in May/June 2019 and then every May/June for the life of the specification.

All materials are available in English only.

Our A-level exams in Further Mathematics include questions that allow students to demonstrate their ability to:

- recall information
- draw together information from different areas of the specification
- apply their knowledge and understanding in practical and theoretical contexts.

4.1 Aims

Courses based on this specification must encourage students to:

- understand mathematics and mathematical processes in ways that promote confidence, foster enjoyment and provide a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding

- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

4.2 Assessment objectives

Assessment objectives (AOs) are set by Ofqual and are the same across all A-level Further Mathematics specifications and all exam boards.

The exams will measure how students have achieved the following assessment objectives.

- AO1: Use and apply standard techniques. Students should be able to:
  - select and correctly carry out routine procedures
  - accurately recall facts, terminology and definitions.
- AO2: Reason, interpret and communicate mathematically. Students should be able to:
  - construct rigorous mathematical arguments (including proofs)
  - make deductions and inferences
  - assess the validity of mathematical arguments
  - explain their reasoning
  - use mathematical language and notation correctly.
- Where questions/tasks targeting this assessment objective will also credit students for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and in other contexts' (AO3) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).
- AO3: Solve problems within mathematics and in other contexts. Students should be able to:
  - translate problems in mathematical and non-mathematical contexts into mathematical processes
  - interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations
  - translate situations in context into mathematical models
  - use mathematical models
  - evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.
- Where questions/tasks targeting this assessment objective will also credit students for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task must be attributed to the corresponding assessment objective(s).

4.2.1 Assessment objective weightings for A-level Further Mathematics

Assessment objectives (AOs)	Component weightings (approx %)			Overall weighting (approx %)
	Paper 1	Paper 2	Paper 3	
AO1	55	55	40	50
AO2	25	25	25	25
AO3	20	20	35	25
Overall weighting of components	33 ⅓	33 ⅓	33 ⅓	100

4.3 Assessment weightings

The marks awarded on the papers will be scaled to meet the weighting of the components. Students' final marks will be calculated by adding together the scaled marks for each component. Grade boundaries will be set using this total scaled mark. The scaling and total scaled marks are shown in the table below.

Students' final marks will be calculated by adding together the scaled marks for each component, this includes the two optional topics chosen as part of paper 3. At qualification level different grade boundaries will be published to reflect the different routes through the qualification.

Component	Maximum raw mark	Scaling factor	Maximum scaled mark
Paper 1	100	x1	100
Paper 2	100	x1	100
Paper 3	100	x1	100
		Total scaled mark:	300

5 General administration

You can find information about all aspects of administration, as well as all the forms you need, at aqa.org.uk/examsadmin

5.1 Entries and codes

You only need to make one entry for each qualification – this will cover all the question papers, non-exam assessment and certification.

Every specification is given a national discount (classification) code by the Department for Education (DfE), which indicates its subject area.

If a student takes two specifications with the same discount code, further and higher education providers are likely to take the view that they have only achieved one of the two qualifications. Please check this before your students start their course.

Qualification title	AQA entry code	DfE discount code
AQA Advanced Level GCE in Further Mathematics	7367	2330

This specification complies with:

- Ofqual *General conditions of recognition* that apply to all regulated qualifications
- Ofqual GCE qualification level conditions that apply to all GCEs
- Ofqual GCE subject level conditions that apply to all GCEs in this subject
- all other relevant regulatory documents.

The Ofqual qualification accreditation number (QAN) is 603/1841/7.

5.2 Overlaps with other qualifications

There is overlapping content in the AS and A-level Further Mathematics specifications. This helps you teach the AS and A-level together.

5.3 Awarding grades and reporting results

The A-level qualification will be graded on a six-point scale: A*, A, B, C, D and E.

Students who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

5.4 Re-sits and shelf life

Students can re-sit the qualification as many times as they wish, within the shelf life of the qualification.

5.5 Previous learning and prerequisites

There are no previous learning requirements. Any requirements for entry to a course based on this specification are at the discretion of schools and colleges.

However, we recommend that students should have the skills and knowledge associated with a GCSE Mathematics or equivalent.

5.6 Access to assessment: diversity and inclusion

General qualifications are designed to prepare students for a wide range of occupations and further study. Therefore our qualifications must assess a wide range of competences.

The subject criteria have been assessed to see if any of the skills or knowledge required present any possible difficulty to any students, whatever their ethnic background, religion, sex, age, disability or sexuality. Tests of specific competences were only included if they were important to the subject.

As members of the Joint Council for Qualifications (JCQ) we participate in the production of the JCQ document *Access Arrangements and Reasonable Adjustments: General and Vocational qualifications*. We follow these guidelines when assessing the needs of individual students who may require an access arrangement or reasonable adjustment. This document is published at jcq.org.uk

Students with disabilities and special needs

We're required by the Equality Act 2010 to make reasonable adjustments to remove or lessen any disadvantage that affects a disabled student.

We can make arrangements for disabled students and students with special needs to help them access the assessments, as long as the competences being tested aren't changed. Access arrangements must be agreed **before** the assessment. For example, a Braille paper would be a reasonable adjustment for a Braille reader.

To arrange access arrangements or reasonable adjustments, you can apply using the online service at aqa.org.uk/eaqa

Special consideration

We can give special consideration to students who have been disadvantaged at the time of the assessment through no fault of their own – for example a temporary illness, injury or serious problem such as family bereavement. We can only do this **after** the assessment.

Your exams officer should apply online for special consideration at aqa.org.uk/eaqa

For more information and advice visit aqa.org.uk/access or email accessarrangementsqueries@aqa.org.uk

5.7 Working with AQA for the first time

If your school or college hasn't previously offered our specifications, you need to register as an AQA centre. Find out how at aqa.org.uk/becomeacentre

5.8 Private candidates

This specification is available to private candidates.

A private candidate is someone who enters for exams through an AQA approved school or college but is not enrolled as a student there.

A private candidate may be self-taught, home schooled or have private tuition, either with a tutor or through a distance learning organisation. They must be based in the UK.

If you have any queries as a private candidate, you can:

- speak to the exams officer at the school or college where you intend to take your exams
- visit our website at aqa.org.uk/privatecandidates
- email privatecandidates@aqa.org.uk

5.9 Use of calculators

A calculator is required for use in all assessments in this specification. Details of the requirements for calculators can be found in the Joint Council for General Qualifications document *Instructions for conducting examinations*.

For A-level Further Mathematics exams, calculators should have the following as a required minimum:

- an iterative function
- the ability to perform calculations with matrices up to order 3 × 3
- the ability to compute summary statistics and access probabilities from standard statistical distributions.

For the purposes of this specification, a ‘calculator’ is any electronic or mechanical device which may be used for the performance of mathematical computations. However, only those permissible in the guidance in the *Instructions for conducting examinations* are allowed in A-level Further Mathematics exams.

6 Appendix A: mathematical notation

The tables below set out the notation that must be used by AS and A-level mathematics and further mathematics specifications. Students will be expected to understand this notation without need for further explanation.

Mathematics students will not be expected to understand notation that relates only to further mathematics content. Further mathematics students will be expected to understand all notation in the list.

For further mathematics, the notation for the core content is listed under sub headings indicating ‘further mathematics only’. In this subject, awarding organisations are required to include, in their specifications, content that is additional to the core content. They will therefore need to add to the notation list accordingly.

AS students will be expected to understand notation that relates to AS content, and will not be expected to understand notation that relates only to A-level content.

6.1 Set notation

1	Set notation	Meaning
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x: \ \ldots\}$	the set of all x such that …
1.7	$n(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$

1	Set notation	Meaning
1.13	$\mathbb{Z}^+$	the set of positive integers $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
1.22	$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$

Set notation (Further Maths only)

1	Set notation	Meaning
1.24	$\mathbb{C}$	the set of complex numbers

6.2 Miscellaneous symbols

2	Miscellaneous symbols	Meaning
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to

2	Miscellaneous symbols	Meaning
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \leq$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term of an arithmetic or geometric sequence
2.17	l	last term of an arithmetic sequence
2.18	d	common difference of an arithmetic sequence
2.19	r	common ratio of a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence

Miscellaneous symbols (Further Maths only)

2	Miscellaneous symbols	Meaning
2.22	$\cong$	is isomorphic to

6.3 Operations

3	Operations	Meaning
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b

3	Operations	Meaning
3.3	$a \times b$, ab, $a.b$	a multiplied by b
3.4	$a \div b$, $\frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\|a\|$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1, n \in \mathbb{N}; 0! = 1$
3.10	$\binom{n}{r}$, ${}^{n}C_{r}$, ${}_{n}C_{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leqslant n$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$

Operations (Further Maths only)

3	Operations	Meaning
3.11	$a \times_n b$	multiplication modulo n of a by b
3.12	$a +_n b$	addition modulo n of a and b
3.13	$G = (<n>, *)$	n is the generator of a given group G under the operation *

6.4 Functions

4	Functions	Meaning
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y

4	Functions	Meaning
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x, \delta x$	an increment of x
4.7	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, ..., nth derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x}, \ddot{x}, \ldots$	the first, second, ... derivatives of x with respect to t
4.11	$\int y \ \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y \ \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$

6.5 Exponential and logarithmic functions

5	Exponential and logarithmic functions	Meaning
5.1	e	base of natural logarithms
5.2	$\mathrm{e}^x, \ \exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x, \log_{\mathrm{e}} x$	natural logarithm of x

6.6 Trigonometric functions

6	Trigonometric functions	Meaning
6.1	sin, cos, tan, cosec, sec, cot	the trigonometric functions
6.2	$\sin^{-1}, \cos^{-1}, \tan^{-1}$ arcsin, arccos, arctan	the inverse trigonometric functions
6.3	°	degrees
6.4	rad	radians

Trigonometric functions (Further Maths only)

6	Trigonometric functions	Meaning
6.5	$\text{cosec}^{-1}, \sec^{-1}, \cot^{-1}$, arccosec, arcsec, arccot	the inverse trigonometric functions
6.6	sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions
6.7	$\sinh^{-1}, \cosh^{-1}, \tanh^{-1}$ $\text{cosech}^{-1}, \text{sech}^{-1}, \coth^{-1}$ arcsinh, arccosh, arctanh, arccosech, arcsech, arccoth	the inverse hyperbolic functions

6.7 Complex numbers (Further Maths only)

7	Complex numbers	Meaning
7.1	i , j	square root of –1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$

7	Complex numbers	Meaning
7.5	Re(z)	the real part of z, Re(z) = x
7.6	Im(z)	the imaginary part of z, Im(z) = y
7.7	$\|z\|$	the modulus of z, $\|z\| = r = \sqrt{x^2 + y^2}$
7.8	arg(z)	the argument of z, arg(z) = θ, $-\pi < \theta \le \pi$
7.9	z^*	the complex conjugate of z, $x - \mathrm{i}y$

Matrices (Further Maths only)

8	Matrices	Meaning
8.1	**M**	a matrix **M**
8.2	**0**	zero matrix
8.3	I	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix **M**
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix **M**
8.6	Δ, det **M** or $\|\mathbf{M}\|$	the determinant of the square matrix **M**
8.7	**Mr**	image of column vector **r** under the transformation associated with the matrix **M**

6.9 Vectors

9	Vectors	Meaning
9.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector **a**, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{a}$	a unit vector in the direction of **a**
9.4	**i**, **j**, **k**	unit vectors in the directions of the cartesian coordinate axes

9	Vectors	Meaning
9.5	$\|\mathbf{a}\|, a$	the magnitude of **a**
9.6	$\|\overrightarrow{AB}\|, AB$	the magnitude of $\overrightarrow{AB}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}, a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.10	$\mathbf{v}$	velocity vector
9.11	$\mathbf{a}$	acceleration vector

Vectors (Further Maths only)

9	Vectors	Meaning
9.12	$\mathbf{a} \cdot \mathbf{b}$	the scalar product of **a** and **b**

6.10 Differential equations (Further Maths only)

10	Differential equations	Meaning
10.1	ω	angular speed

6.11 Probability and statistics

11	Probability and statistics	Meaning
11.1	A, B, C etc.	events
11.2	$A \cup B$	union of the events A and B
11.3	$A \cap B$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A
11.5	A'	complement of the event A
11.6	$\mathrm{P}(A \| B)$	probability of the event A conditional on the event B

11	Probability and statistics	Meaning
11.7	X, Y, R etc.	random variables
11.8	x, y, r etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \dots$	values of observations
11.10	$f_1, f_2, \dots$	frequencies with which the observations $x_1, x_2, \dots$ occur
11.11	$\mathrm{p}(x), \mathrm{P}(X = x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \dots$	probabilities of the values $x_1, x_2, \dots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$Z \sim \mathrm{N}(0,1)$	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution N(0,1)
11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation

11	Probability and statistics	Meaning
11.28	H_0	null hypothesis
11.29	H_1	alternative hypothesis
11.30	r	product moment correlation coefficient for a sample
11.31	ρ	product moment correlation coefficient for a population

6.12 Mechanics

12	Mechanics	Meaning
12.1	kg	kilogram
12.2	m	metre
12.3	km	kilometre
12.4	m/s, $\mathrm{m\ s^{-1}}$	metre(s) per second (velocity)
12.5	$\mathrm{m/s^2}$, $\mathrm{m\ s^{-2}}$	metre(s) per second per second (acceleration)
12.6	F	Force or resultant force
12.7	N	newton
12.8	Nm	newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction

7 Appendix B: mathematical formulae and identities

Students must be able to use the following formulae and identities for AS and A-level further mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure mathematics

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a(xy)$

$\log_a x - \log_a y \equiv \log_a\left(\frac{x}{y}\right)$

$k\log_a x \equiv \log_a(x^k)$

Coordinate geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression: $u_n = a + (n - 1)d$

General term of a geometric progression: $u_n = ar^{n-1}$

Trigonometry

In the triangle *ABC*

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area = $\frac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\text{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and Area of circle, radius r and diameter d :

$C = 2\pi r = \pi d$

$A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a , b and c are the lengths of the sides and c is the hypotenuse:

$c^2 = a^2 + b^2$

Area of a trapezium = $\frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r , where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$s = r\theta$

$A = \frac{1}{2}r^2\theta$

Complex numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 + \theta_2)}$

Loci in the Argand diagram:

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis.

Exponential form:

$e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrices **A**, **B**:

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Algebra

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

For $ax^2 + bx + c = 0$ with roots α and β :

$\alpha + \beta = \frac{-b}{a}$

$\alpha\beta = \frac{c}{a}$

For $ax^3 + bx^2 + cx + d = 0$ with roots α , β and γ :

$\sum\alpha = \frac{-b}{a}$

$\sum\alpha\beta = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-d}{a}$

Hyperbolic functions

$\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$

$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$

$\tanh x \equiv \frac{\sinh x}{\cosh x}$

Calculus and differential equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$

Area under a curve = $\int_a^b y \ \mathrm{d}x \ (y \geq 0)$

Volumes of revolution about the x and y axes:

$$V_x = \pi \int_a^b y^2 \mathrm{d}x$$

$$V_y = \pi \int_c^d x^2 \mathrm{d}y$$

Simple Harmonic Motion:

$$\ddot{x} = -\omega^2 x$$

Vectors

$$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{(x^2 + y^2 + z^2)}$$

Scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$$

where θ is the acute angle between the vectors **a** and **b**.

The equation of the line through the point with position vector **a** parallel to vector **b** is:

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

The equation of the plane containing the point with position vector **a** and perpendicular to vector **n** is:

$$(\mathbf{r} - \mathbf{a}) . \mathbf{n} = 0$$

Mechanics

Forces and equilibrium

Weight = mass × g

Friction: $F \leq \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v = \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$$

$$r = \int v \ \mathrm{d}t$$

$$v = \int a \ \mathrm{d}t$$

Statistics

The mean of a set of data: $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \frac{X - \mu}{\sigma}$ where $X \sim \mathrm{N}(\mu, \sigma^2)$

AQA

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